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Reference

Properties

Properties of Exponents

Let a and b be real numbers and let m and n be rational numbers.

Zero Exponent

$$a^0 = 1, \text{ where } a \neq 0$$

Negative Exponent

$$a^{-n} = \frac{1}{a^n}, \text{ where } a \neq 0$$

Product of Powers Property

$$a^m \cdot a^n = a^{m+n}$$

Quotient of Powers Property

$$\frac{a^m}{a^n} = a^{m-n}, \text{ where } a \neq 0$$

Power of a Power Property

$$(a^m)^n = a^{mn}$$

Power of a Product Property

$$(ab)^m = a^m b^m$$

Power of a Quotient Property

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}, \text{ where } b \neq 0$$

Rational Exponents

$$a^{m/n} = (a^{1/n})^m = (\sqrt[n]{a})^m$$

Rational Exponents

$$a^{-m/n} = \frac{1}{a^{m/n}} = \frac{1}{(a^{1/n})^m} = \frac{1}{(\sqrt[n]{a})^m},$$

where $a \neq 0$

Properties of Radicals

Let a and b be real numbers and let n be an integer greater than 1.

Product Property of Radicals

$$\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$$

Quotient Property of Radicals

$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}, \text{ where } a \geq 0 \text{ and } b \neq 0$$

Square Root of a Negative Number

1. If r is a positive real number, then $\sqrt{-r} = i\sqrt{r}$.
2. By the first property, it follows that $(i\sqrt{r})^2 = -r$.

Properties of Logarithms

Let b , m , and n be positive real numbers with $b \neq 1$.

Product Property

$$\log_b mn = \log_b m + \log_b n$$

Quotient Property

$$\log_b \frac{m}{n} = \log_b m - \log_b n$$

Power Property

$$\log_b m^n = n \log_b m$$

Other Properties

Zero-Product Property

If A and B are expressions and $AB = 0$, then $A = 0$ or $B = 0$.

Property of Equality for Exponential Equations

If $b > 0$ and $b \neq 1$, then $b^x = b^y$ if and only if $x = y$.

Property of Equality for Logarithmic Equations

If b , x , and y are positive real numbers with $b \neq 1$, then $\log_b x = \log_b y$ if and only if $x = y$.

Patterns

Square of a Binomial Pattern

$$(a + b)^2 = a^2 + 2ab + b^2$$

$$(a - b)^2 = a^2 - 2ab + b^2$$

Cube of a Binomial

$$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$$

Difference of Two Squares Pattern

$$a^2 - b^2 = (a + b)(a - b)$$

Sum of Two Cubes

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

Sum and Difference Pattern

$$(a + b)(a - b) = a^2 - b^2$$

Completing the Square

$$x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2$$

Perfect Square Trinomial Pattern

$$a^2 + 2ab + b^2 = (a + b)^2$$

$$a^2 - 2ab + b^2 = (a - b)^2$$

Difference of Two Cubes

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

Theorems

The Remainder Theorem

If a polynomial $f(x)$ is divided by $x - k$, then the remainder is $r = f(k)$.

The Factor Theorem

A polynomial $f(x)$ has a factor $x - k$ if and only if $f(k) = 0$.

The Rational Root Theorem

If $f(x) = a_nx^n + \cdots + a_1x + a_0$ has *integer* coefficients, then every rational solution of $f(x) = 0$ has the form

$$\frac{p}{q} = \frac{\text{factor of constant term } a_0}{\text{factor of leading coefficient } a_n}.$$

The Irrational Conjugates Theorem

Let f be a polynomial function with rational coefficients, and let a and b be rational numbers such that $\sqrt{b}$ is irrational.

If $a + \sqrt{b}$ is a zero of f , then $a - \sqrt{b}$ is also a zero of f .

The Fundamental Theorem of Algebra

Theorem If $f(x)$ is a polynomial of degree n where $n > 0$, then the equation $f(x) = 0$ has at least one solution in the set of complex numbers.

Corollary If $f(x)$ is a polynomial of degree n where $n > 0$, then the equation $f(x) = 0$ has exactly n solutions provided each solution repeated twice is counted as 2 solutions, each solution repeated three times is counted as 3 solutions, and so on.

The Complex Conjugates Theorem

If f is a polynomial function with real coefficients, and $a + bi$ is an imaginary zero of f , then $a - bi$ is also a zero of f .

Descartes's Rule of Signs

Let $f(x) = a_nx^n + a_{n-1}x^{n-1} + \cdots + a_2x^2 + a_1x + a_0$ be a polynomial function with real coefficients.

- The number of positive real zeros of f is equal to the number of changes in sign of the coefficients of $f(x)$ or is less than this by an even number.
- The number of negative real zeros of f is equal to the number of changes in the sign of the coefficients of $f(-x)$ or is less than this by an even number.

Formulas

Algebra

Slope

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Slope-intercept form

$$y = mx + b$$

Point-slope form

$$y - y_1 = m(x - x_1)$$

Standard form of a quadratic function

$$f(x) = ax^2 + bx + c, \text{ where } a \neq 0$$

Intercept form of a quadratic function

$$f(x) = a(x - p)(x - q), \text{ where } a \neq 0$$

Standard equation of a circle

$$x^2 + y^2 = r^2$$

Exponential growth function

$$y = ab^x, \text{ where } a \neq 0 \text{ and } b > 1$$

Logarithm of y with base b

$$\log_b y = x \text{ if and only if } b^x = y$$

Sum of n terms of 1

$$\sum_{i=1}^n 1 = n$$

Sum of squares of first n positive integers

$$\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$$

Sum of first n terms of an arithmetic series

$$S_n = n \frac{(a_1 + a_n)}{2}$$

Sum of first n terms of a geometric series

$$S_n = a_1 \left(\frac{1 - r^n}{1 - r} \right), \text{ where } r \neq 1$$

Recursive equation for an arithmetic sequence

$$a_n = a_{n-1} + d$$

Statistics

Sample mean

$$\bar{x} = \frac{\sum x}{n}$$

z-Score

$$z = \frac{x - \mu}{\sigma}$$

Vertex form of a quadratic function

$$f(x) = a(x - h)^2 + k, \text{ where } a \neq 0$$

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \text{ where } a \neq 0$$

Standard form of a polynomial function

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

Exponential decay function

$$y = ab^x, \text{ where } a \neq 0 \text{ and } 0 < b < 1$$

Change-of-base formula

$$\log_c a = \frac{\log_b a}{\log_b c}, \text{ where } a, b, \text{ and } c \text{ are positive real numbers with } b \neq 1 \text{ and } c \neq 1.$$

Sum of first n positive numbers

$$\sum_{i=1}^n i = \frac{n(n+1)}{2}$$

Explicit rule for an arithmetic sequence

$$a_n = a_1 + (n - 1)d$$

Explicit rule for a geometric sequence

$$a_n = a_1 r^{n-1}$$

Sum of an infinite geometric series

$$S = \frac{a_1}{1 - r} \text{ provided } |r| < 1$$

Recursive equation for a geometric sequence

$$a_n = r \cdot a_{n-1}$$

Standard deviation

$$\sigma = \sqrt{\frac{(x_1 - \mu)^2 + (x_2 - \mu)^2 + \cdots + (x_n - \mu)^2}{n}}$$

Margin of error for sample proportions

$$\pm \frac{1}{\sqrt{n}}$$

Trigonometry

General definitions of trigonometric functions

Let θ be an angle in standard position, and let (x, y) be the point where the terminal side of θ intersects the circle $x^2 + y^2 = r^2$. The six trigonometric functions of θ are defined as shown.

$$\sin \theta = \frac{y}{r}$$

$$\cos \theta = \frac{x}{r}$$

$$\tan \theta = \frac{y}{x}, x \neq 0$$

$$\csc \theta = \frac{r}{y}, y \neq 0$$

$$\sec \theta = \frac{r}{x}, x \neq 0$$

$$\cot \theta = \frac{x}{y}, y \neq 0$$

Conversion between degrees and radians

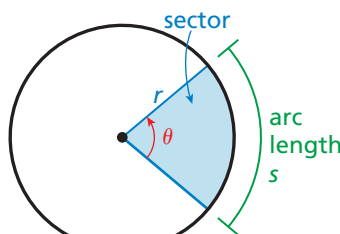
$$180^\circ = \pi \text{ radians}$$

Arc length of a sector

$$s = r\theta$$

Area of a sector

$$A = \frac{1}{2}r^2\theta$$



Reciprocal Identities

$$\csc \theta = \frac{1}{\sin \theta}$$

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\cot \theta = \frac{1}{\tan \theta}$$

Tangent and Cotangent Identities

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

Pythagorean Identities

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$1 + \tan^2 \theta = \sec^2 \theta$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

Negative Angle Identities

$$\sin(-\theta) = -\sin \theta$$

$$\cos(-\theta) = \cos \theta$$

$$\tan(-\theta) = -\tan \theta$$

Cofunction Identities

$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos \theta$$

$$\cos\left(\frac{\pi}{2} - \theta\right) = \sin \theta$$

$$\tan\left(\frac{\pi}{2} - \theta\right) = \cot \theta$$

Sum Formulas

$$\sin(a + b) = \sin a \cos b + \cos a \sin b$$

$$\cos(a + b) = \cos a \cos b - \sin a \sin b$$

$$\tan(a + b) = \frac{\tan a + \tan b}{1 - \tan a \tan b}$$

Difference Formulas

$$\sin(a - b) = \sin a \cos b - \cos a \sin b$$

$$\cos(a - b) = \cos a \cos b + \sin a \sin b$$

$$\tan(a - b) = \frac{\tan a - \tan b}{1 + \tan a \tan b}$$

Probability and Combinatorics

$$\text{Theoretical Probability} = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}}$$

$$\text{Experimental Probability} = \frac{\text{Number of successes}}{\text{Number of trials}}$$

Probability of the complement of an event

$$P(\bar{A}) = 1 - P(A)$$

Probability of independent events

$$P(A \text{ and } B) = P(A) \cdot P(B)$$

Probability of dependent events

$$P(A \text{ and } B) = P(A) \cdot P(B | A)$$

Probability of compound events

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

Permutations

$${}_nP_r = \frac{n!}{(n-r)!}$$

Combinations

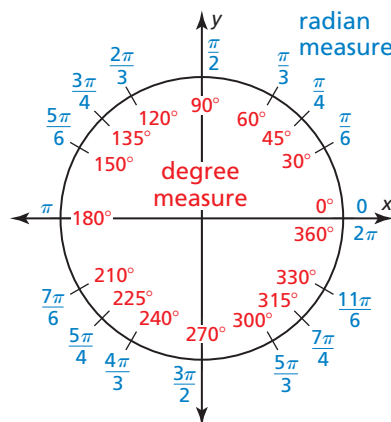
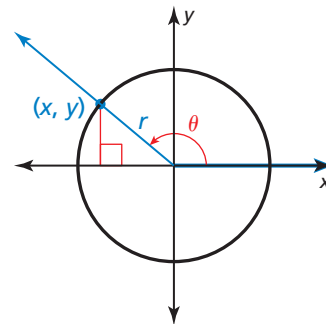
$${}_nC_r = \frac{n!}{(n-r)! \cdot r!}$$

Binomial experiments

$$P(k \text{ successes}) = {}_nC_k p^k (1-p)^{n-k}$$

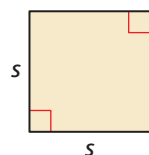
The Binomial Theorem

$$(a + b)^n = {}_nC_0 a^n b^0 + {}_nC_1 a^{n-1} b^1 + {}_nC_2 a^{n-2} b^2 + \cdots + {}_nC_n a^0 b^n, \text{ where } n \text{ is a positive integer.}$$



Perimeter, Area, and Volume Formulas

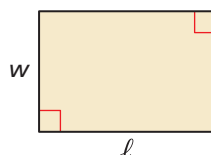
Square



$$P = 4s$$

$$A = s^2$$

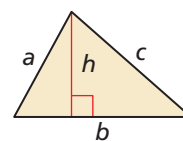
Rectangle



$$P = 2\ell + 2w$$

$$A = \ell w$$

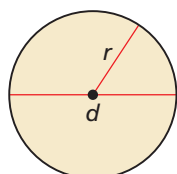
Triangle



$$P = a + b + c$$

$$A = \frac{1}{2}bh$$

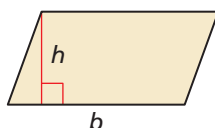
Circle



$$C = \pi d \text{ or } C = 2\pi r$$

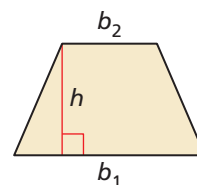
$$A = \pi r^2$$

Parallelogram



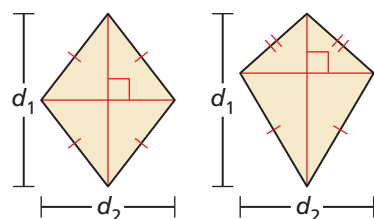
$$A = bh$$

Trapezoid



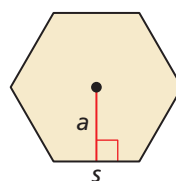
$$A = \frac{1}{2}h(b_1 + b_2)$$

Rhombus/Kite



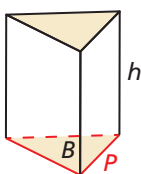
$$A = \frac{1}{2}d_1d_2$$

Regular n-gon



$$A = \frac{1}{2}aP \text{ or } A = \frac{1}{2}a \cdot ns$$

Prism

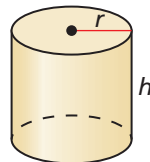


$$L = Ph$$

$$S = 2B + Ph$$

$$V = Bh$$

Cylinder

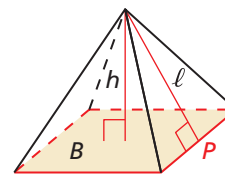


$$L = 2\pi rh$$

$$S = 2\pi r^2 + 2\pi rh$$

$$V = \pi r^2 h$$

Pyramid

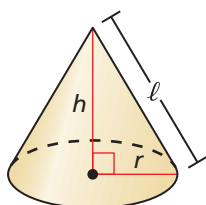


$$L = \frac{1}{2}P\ell$$

$$S = B + \frac{1}{2}P\ell$$

$$V = \frac{1}{3}Bh$$

Cone

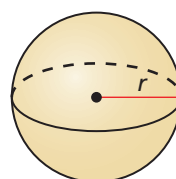


$$L = \pi r\ell$$

$$S = \pi r^2 + \pi r\ell$$

$$V = \frac{1}{3}\pi r^2 h$$

Sphere



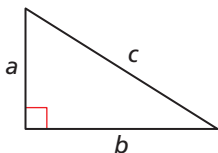
$$S = 4\pi r^2$$

$$V = \frac{4}{3}\pi r^3$$

Other Formulas

Pythagorean Theorem

$$a^2 + b^2 = c^2$$



Simple Interest

$$I = Prt$$

Distance

$$d = rt$$

Compound Interest

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

Continuously Compounded Interest

$$A = Pe^{rt}$$

Conversions

U.S. Customary

- 1 foot = 12 inches
- 1 yard = 3 feet
- 1 mile = 5280 feet
- 1 mile = 1760 yards
- 1 acre = 43,560 square feet
- 1 cup = 8 fluid ounces
- 1 pint = 2 cups
- 1 quart = 2 pints
- 1 gallon = 4 quarts
- 1 gallon = 231 cubic inches
- 1 pound = 16 ounces
- 1 ton = 2000 pounds

U.S. Customary to Metric

- 1 inch = 2.54 centimeters
- 1 foot $\approx$ 0.3 meter
- 1 mile $\approx$ 1.61 kilometers
- 1 quart $\approx$ 0.95 liter
- 1 gallon $\approx$ 3.79 liters
- 1 cup $\approx$ 237 milliliters
- 1 pound $\approx$ 0.45 kilogram
- 1 ounce $\approx$ 28.3 grams
- 1 gallon $\approx$ 3785 cubic centimeters

Time

- 1 minute = 60 seconds
- 1 hour = 60 minutes
- 1 hour = 3600 seconds
- 1 year = 52 weeks

Temperature

$$C = \frac{5}{9}(F - 32)$$
$$F = \frac{9}{5}C + 32$$

Metric

- 1 centimeter = 10 millimeters
- 1 meter = 100 centimeters
- 1 kilometer = 1000 meters
- 1 liter = 1000 milliliters
- 1 kiloliter = 1000 liters
- 1 milliliter = 1 cubic centimeter
- 1 liter = 1000 cubic centimeters
- 1 cubic millimeter = 0.001 milliliter
- 1 gram = 1000 milligrams
- 1 kilogram = 1000 grams

Metric to U.S. Customary

- 1 centimeter $\approx$ 0.39 inch
- 1 meter $\approx$ 3.28 feet
- 1 meter $\approx$ 39.37 inches
- 1 kilometer $\approx$ 0.62 mile
- 1 liter $\approx$ 1.06 quarts
- 1 liter $\approx$ 0.26 gallon
- 1 kilogram $\approx$ 2.2 pounds
- 1 gram $\approx$ 0.035 ounce
- 1 cubic meter $\approx$ 264 gallons